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World-volume potentials on D-branes

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Abstract: By evaluating string scattering amplitudes, we investigate various low energy interactions for the massless scalars on a non-abelian Dirichlet brane. We confirm the existence of couplings of closed string fields to the world-volume scalars, involving commutators of the latter. Our results are consistent with the recently proposed non-abelian world-volume actions for Dp-branes.

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Contents

1. Introduction

D-branes have proven to be invaluable tools in investigating non-perturbative properties of string theory — see, e.g. [1]. While originally conceived as simply surfaces supporting open strings [2], Polchinski later elucidated their role as dynamical stringy solitons carrying Ramond-Ramond charges [3]. The low energy action describing the dynamics of test D-branes consists of two parts: the Born-Infeld action, which provides the kinetic terms for the world-volume fields, and also contains the couplings of the D-brane to the massless Neveu-Schwarz (NS) fields in the bulk supergravity [4]; and the Wess-Zumino action, which contains the couplings to the massless Ramond-Ramond (RR) fields [5, 6]. This non-linear world-volume action reliably describes the physics of D-branes with surprising accuracy.

One remarkable aspect of the D-brane story is that the U(1) gauge symmetry of an individual D-brane is enhanced to a non-abelian U(N) symmetry for N coincident Dp-branes [7]. The form of the action for non-abelian D-branes in general background fields was recently given in refs. [8, 9]. Just as for the abelian theory of an individual D-brane, the non-abelian action describing multiple D-branes has two distinct pieces: the Born-Infeld action

$$S_{\rm BI} = -T_p \int d^{p+1}\sigma \times \qquad (1.1)$$
$$\times\ \mathrm{STr}\left(e^{-\phi}\sqrt{-\det\left(P\left[E_{ab}+E_{ai}(Q^{-1}-\delta)^{ij}E_{jb}\right]+\lambda\,F_{ab}\right)\,\det(Q^i{}_j)}\right),$$

with

$$E_{\mu\nu} = G_{\mu\nu} + B_{\mu\nu}\,, \qquad \text{and} \qquad Q^i{}_j \equiv \delta^i{}_j + i\lambda\,[\Phi^i, \Phi^k]\,E_{kj}\,, \tag{1.2}$$

and the Wess-Zumino action

$$S_{\rm WZ} = \mu_p \int \mathrm{STr}\left(P\left[e^{i\lambda\,\mathrm{i}_\Phi \mathrm{i}_\Phi}\left(\sum C^{(n)}\,e^B\right)\right]e^{\lambda F}\right). \tag{1.3}$$

In both of these expressions, $\lambda = 2\pi\ell_s^2$. We refer the interested reader to ref. [8] for more details on these actions and our notation.

The displacements of the branes in the transverse space are parameterized by the world-volume scalar fields, Φ^i, $i = p+1, \ldots, 9$. In the non-abelian theory, however, the scalars are in the adjoint representation of the U(N) world-volume gauge symmetry. These scalars appear in the action in three ways: first, there is the explicit appearance in the first exponential in eq. (1.3). Here, i_Φ denotes the interior product by Φ^i regarded as a vector in the transverse space, e.g. acting on an n-form $C^{(n)} = \frac{1}{n!} C^{(n)}_{\mu_1\cdots\mu_n} dx^{\mu_1}\cdots dx^{\mu_n}$, we have

$$\mathrm{i}_\Phi \mathrm{i}_\Phi C^{(n)} = \frac{1}{2(n-2)!}[\Phi^i, \Phi^j]\,C^{(n)}_{ji\mu_3\cdots\mu_n} dx^{\mu_3}\cdots dx^{\mu_n}\,.$$

Next, both actions involve the pull-back (denoted by $P[\cdots]$) of various spacetime tensors to the world-volume which now involves covariant derivatives of the non-abelian scalars. For example,

$$P[E]_{ab} = E_{ab} + \lambda\,E_{ai}\,D_b\Phi^i + \lambda\,E_{ib}\,D_a\Phi^i + \lambda^2\,E_{ij} D_a\Phi^i D_b\Phi^j\,.$$

Finally, the bulk supergravity fields are in general functions of all of the spacetime coordinates, and so in the world-volume action, they are implicitly functionals of the non-abelian scalars. For example, the metric functional appearing in the D-brane action would be given by a non-abelian Taylor expansion

$$\begin{aligned} G_{\mu\nu} &= \exp\left[\lambda\Phi^i\,\partial_{x^i}\right] G^0_{\mu\nu}(\sigma^a, x^i)|_{x^i=0} \\ &= \sum_{n=0}^{\infty} \frac{\lambda^n}{n!}\,\Phi^{i_1}\cdots\Phi^{i_n}\,(\partial_{x^{i_1}}\cdots\partial_{x^{i_n}}) G^0_{\mu\nu}(\sigma^a, x^i)|_{x^i=0}\,. \end{aligned}$$

In both parts of the action, eqs. (1.1) and (1.3), there is a single (symmetrized) gauge trace which encompasses all of the scalars appearing in these various ways (and, of course, the gauge fields as well).

The implicit appearance of the non-abelian scalars in the functional dependence [10] and pull-back [11] of the background fields was originally argued on general grounds. Both of these suggestions can be confirmed to leading order by examining string scattering amplitudes [12]. The interesting commutator couplings to the bulk fields were first discovered in the construction of the non-abelian action in

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refs. [8, 9] — see also [13]. There, the action was deduced by demanding that the non-abelian theory must be consistent with T-duality. In the present paper, we confirm the appearance of the new commutator interactions in the non-abelian Dp-brane action by the direct examination of string scattering amplitudes.

The remainder of the paper is organized as follows: in section 2, we consider the world-volume field theory and identify an interesting set of string scattering amplitudes. In section 3, we evaluate two types of amplitudes describing the scattering of: (i) three scalars and a RR field, and (ii) two world-volume scalars, a gauge boson and a RR field. Comparing these results with the interactions expected in the action (1.3), we find precise agreement. In section 4, we consider a similar set of amplitudes where the closed string field is the Neveu-Schwarz two-form, rather than a RR field. We conclude in section 5 with some further discussion of our results. In appendix A, we describe the details of evaluating the integrals necessary to calculate the desired scattering amplitudes.

2. Low energy field theory on the world-volume

We are interested in finding evidence of various commutator interactions appearing in the world-volume action. Clearly the U(1) components of the world-volume fields will not participate in these interactions. Further, any commutator will itself be in the adjoint of SU(N), and so have a vanishing trace. To produce a non-trivial interaction then, we will need at least three world-volume fields in the interaction. Hence the scattering amplitudes, which we must evaluate, will involve at least three open string states and a single closed string state. Such an amplitude is equivalent to a five-point open string amplitude with unusual kinematics [14, 15]. While these amplitudes can be evaluated (see, e.g. [16]), it will be a fairly laborious exercise. Here, we will examine the amplitudes of interest within the world-volume field theory before proceeding with the string scattering calculations.

To begin let us focus the discussion on the Wess-Zumino action (1.3). In this part of the action, the minimal interaction of interest will involve a bulk RR field and the two world-volume scalars entering in the commutator. For a Dp-brane then, the most straightforward interactions involve the $(p+3)$-form potential

$$\begin{aligned} S^{(i)} &= i\lambda\mu_p \int \mathrm{STr}\left(P\left[\mathrm{i}_\Phi \mathrm{i}_\Phi C^{(p+3)}(\sigma,\Phi)\right]\right) \\ &= \frac{i}{2}\lambda^2\mu_p \int d^{p+1}\sigma \frac{1}{p!}(\varepsilon^v)^{a_0\cdots a_p}\Big[\mathrm{Tr}\left(\left[\Phi^j,\Phi^i\right]\partial_{a_0}\Phi^k\right) C^{(p+3)}_{ijka_1\cdots a_p}(\sigma) + \qquad (2.1) \\ &\qquad\qquad + \frac{1}{p+1}\,\mathrm{Tr}\left([\Phi^j,\Phi^i]\Phi^k\right)\partial_k C^{(p+3)}_{ija_0\cdots a_p}(\sigma) + \cdots\Big]. \end{aligned}$$

In the second line, we have only explicitly kept the leading non-trivial interactions, which involve three world-volume scalars. There, the third scalar arises from the pull-back in the first term and from the non-abelian Taylor expansion in the second term. Note with an integration by parts, these two terms can be combined to yield the following simple result:

$$S^{(i)} = \frac{i}{3}\lambda^2\mu_p \int d^{p+1}\sigma \frac{1}{(p+1)!}(\varepsilon^v)^{a_0\cdots a_p}\ \mathrm{Tr}\left(\Phi^i\Phi^j\Phi^k\right) F^{(p+4)}_{ijka_0\cdots a_p}(\sigma)\,, \qquad (2.2)$$

where $F^{(p+4)} = dC^{(p+3)}$. To verify the presence of this interaction in the low energy effective action, we examine the $\alpha' \to 0$ limit of the string amplitudes involving three scalars and the RR $(p+3)$-form.

Generically in the limit $\alpha' \to 0$, we expect string scattering amplitudes may contain massless poles corresponding to the exchange of massless string states arising from lower order interactions in the effective field theory. That is, the leading low energy terms for the scalar field arise from the Born-Infeld action (1.1)

$$-\lambda^2 T_p \int d^{p+1}\sigma\ \mathrm{Tr}\left(\frac{1}{2}D^a\Phi^i D_a\Phi^i - \frac{1}{4}[\Phi^i,\Phi^j][\Phi^i,\Phi^j]\right), \qquad (2.3)$$

where, with the present conventions, $D_a\Phi^i = \partial_a\Phi^i + i[A_a,\Phi^i]$. Hence the non-abelian scalar field theory includes the usual four-point interaction, and also interactions with the gauge field, e.g. the standard three-point interaction $\mathrm{Tr}(\partial^a\Phi^i\,[A_a,\Phi^i])$. Hence we might expect low energy processes where two of the open string states (in the amplitudes of interest) scatter to emit a massless virtual particle which is absorbed by a lower order world-volume interaction involving the RR field. Such an exchange would be responsible for poles appearing in the string amplitude, and so a field theory calculation must be done to properly subtract out such poles and identify the contact interactions.

However, in fact, for the amplitude involving three scalars and the RR $(p+3)$-form, one finds there are no such contributions involving the exchange of massless particles. Essentially, the $(p+3)$-form potential has too many spacetime indices to produce a lower order interaction in the $(p+1)$-dimensional world-volume theory. Hence we conclude, that in fact, the string amplitude will contain no massless poles. Thus the leading contribution in the $\alpha' \to 0$ limit will be a set of contact terms which we should be able identify as arising from the low energy interaction (2.2).

Next we extend our analysis to consider interactions in the Wess-Zumino action involving a bulk RR field, the two world-volume scalars entering in the commutator, and a world-volume gauge field. For example on a Dp-brane, eq. (1.3) includes

$$\begin{aligned} S^{(ii)} &= i\lambda^2\mu_p \int \mathrm{STr}\left(P\left[\mathrm{i}_\Phi\mathrm{i}_\Phi C^{(p+1)}(\sigma,\Phi)\right] F\right) \qquad (2.4)\\ &= \frac{i}{4}\lambda^2\mu_p \int d^{p+1}\sigma \frac{1}{(p-1)!}(\varepsilon^v)^{a_0\cdots a_p}\ \mathrm{Tr}\left([\Phi^j,\Phi^i]F_{a_0a_1}\right) C^{(p+1)}_{ija_2\cdots a_p}(\sigma) + \cdots\,. \end{aligned}$$

In fact, the analysis of the corresponding amplitude also requires considering interactions which as above are order λ^2 but arise simply from the pull-back or Taylor expansion of $C^{(p+1)}$

$$S^{(iii)} = \frac{\lambda^2\mu_p}{2}\int d^{p+1}\sigma\frac{1}{(p+1)!}(\varepsilon^v)^{a_0\cdots a_p}\left[\mathrm{Tr}\left(\Phi^j\Phi^i\right)\partial_j\partial_i C^{(p+1)}_{a_0\cdots a_p}(\sigma)+\right.$$
$$+2(p+1)\ \mathrm{Tr}\left(\Phi^j D_{a_0}\Phi^i\right)\partial_j C^{(p+1)}_{ia_1\cdots a_p}(\sigma)+$$
$$\left.+p(p+1)\ \mathrm{Tr}\left(D_{a_0}\Phi^i\, D_{a_1}\Phi^j\right)C^{(p+1)}_{ija_2\cdots a_p}(\sigma)\right].$$

Now again after integrating by parts a number of times, these contributions can be rewritten as

$$\begin{aligned}S^{(ii)}+S^{(iii)} &= \frac{\lambda^2}{2}\mu_p\int d^{p+1}\sigma\frac{1}{(p+1)!}(\varepsilon^v)^{a_0\cdots a_p}\left[\mathrm{Tr}\left(\Phi^j\Phi^i\right)\partial_j F^{(p+2)}_{ia_0\cdots a_p}(\sigma)+\right.\\ &\qquad\left.+(p+1)\ \mathrm{Tr}\left(D_{a_0}\Phi^j\Phi^i\right)F^{(p+2)}_{ija_1\cdots a_p}(\sigma)\right]\\ &= \frac{\lambda^2}{2}\mu_p\int d^{p+1}\sigma\frac{1}{(p+1)!}(\varepsilon^v)^{a_0\cdots a_p}\times\\ &\qquad\times\left[\mathrm{Tr}\left(\Phi^j\Phi^i\right)\partial_j F^{(p+2)}_{ia_0\cdots a_p}(\sigma)+(p+1)\ \mathrm{Tr}\left(\partial_{a_0}\Phi^j\Phi^i\right)F^{(p+2)}_{ija_1\cdots a_p}(\sigma)+\right.\\ &\qquad\left.+2i(p+1)\ \mathrm{Tr}\left(A_{a_0}\Phi^j\Phi^i\right)F^{(p+2)}_{ija_1\cdots a_p}(\sigma)\right],\end{aligned}\tag{2.5}$$

where $F^{(p+2)} = dC^{(p+1)}$. In the final expression, the last term gives the desired contact interaction involving (a commutator of) two scalars and a single gauge field. The first two terms are lower order in that they only involve two scalars, but these will be relevant for determining the massless poles that appear in the string amplitude.

In examining the latter, one might consider a virtual gluon A propagating between two lower order interactions. However, as above, one finds that there are no interactions involving the RR $(p+1)$-form which would allow such an exchange. On the other hand, the exchange of a virtual scalar is possible. Here the two-scalar interactions in eq. (2.5) allow a closed string RR potential to interact with an onshell scalar emitting a virtual scalar. The latter is then absorbed through the standard three-point interaction appearing in eq. (2.3) to produce an onshell scalar and gauge field.

Hence we expect to find two massless poles in the string scattering amplitude involving a bulk $C^{(p+1)}$ field, the two scalars Φ and a world-volume gauge field A. To be more specific, let us label the external states as:

$$\begin{aligned}\text{gauge vector:}\quad & A^a_1,\ k_1\\ \text{transverse scalars:}\quad & \Phi^i_2,\ k_2\\ & \Phi^j_3,\ k_3\\ \text{RR }(p+1)\text{-form:}\quad & C^{p+1}_4,\ p_4\,.\end{aligned}$$

For later purposes, we also define

$$s=-2k_1\cdot k_3\,,\qquad t=-2k_1\cdot k_2\,,\qquad u=-2k_2\cdot k_3\,.\tag{2.6}$$

(Note that the component of p_4 orthogonal to the brane is not conserved — see, e.g. ref. [14, 15] — and so $s+t+u=(p_4^\perp)^2$.) With these definitions, the massless poles in the four-point amplitude of interest will be in the s and t channels. With the field theory subtractions, we will be able to read off the leading contact contribution in the string amplitude, which should match the last term in eq. (2.5). In fact the subtraction is simple since a quick examination of the low energy interactions in eqs. (2.3) and (2.5) shows there can be no contributions with canceling factors of contracted momenta appearing in the numerator of these field theory amplitudes, e.g. $k_1\cdot k_2$ over the s channel pole. Hence the field theory subtractions correspond to subtracting the purely pole terms out of the string scattering amplitude.

In principle, one should consider the possible appearance of pole terms of the form $1/(p_4^\perp)^2=1/(s+t+u)$. Such contributions would arise if there was an interaction involving the RR form and a single world-volume field. For the case of interest, i.e. a RR $(p+1)$-form coupling to a Dp-brane, the only such interaction in the Wess-Zumino action (1.3) may be written as

$$S^{(iv)}=\lambda\mu_p\int d^{p+1}\sigma\frac{1}{(p+1)!}(\varepsilon^v)^{a_0\cdots a_p}\ \mathrm{Tr}\left(\Phi^i\right)\ F^{(p+2)}_{ia_0\cdots a_p}(\sigma)\,.$$

Of course, only the U(1) component of the scalar contributes in this interaction. Now in a scattering process, the virtual scalar created by the above interaction would have to be absorbed in an interaction involving three scalars and a gauge field, but there are no world-volume interactions of the desired form. Equation (2.3) does include an interaction with two scalars and two gauge fields, but there is no possibility to exchange a virtual gauge field because it can not be absorbed by a RR $(p+1)$-form. Thus the amplitude of interest should not include any contributions proportional to $1/(s+t+u)$.

Finally we turn our attention to the non-abelian Born-Infeld action (1.1). Non-trivial commutator interactions appear here through the Q matrix (1.2), but as above any non-trivial interaction involving a single commutator must also include at least one other open string field. The simplest case to consider is interactions involving three scalars and the NS fields. After some calculations similar to those above, one finds the following two interactions:

$$S^{(v)}=i\lambda^2T_p\int d^{p+1}\sigma\left[\frac{1}{3}\,\mathrm{Tr}(\Phi^i\Phi^j\Phi^k)H_{ijk}+\mathrm{Tr}(D^a\Phi^j[\Phi^j,\Phi^i])B_{ia}\right],\qquad(2.7)$$

where B is the NS two-form and $H=dB$. The first term here arises from the expansion of the $\det(Q^i{}_j)$ factor, while the second term from the pull-back in the first determinant factor. Note that the first determinant factor in eq. (1.1) contains other interactions involving the graviton or dilaton and three scalars, but they are higher order in λ, i.e. they contain three derivatives. It is also straightforward to show that the field theory predicts there are no massless poles in the corresponding string scattering amplitudes.

3. Ramond-Ramond string amplitudes

The amplitude for a RR closed string state scattering with three open strings consisting of either three scalars or one gauge and two scalars on Dp-brane is given by

$$A_{123} = -\frac{\lambda^2 \mu_p}{2\sqrt{2}\pi} \mathrm{Tr} \int dx_1 dx_2 dx_3 d^2 z_4 \left\langle V_1^{\mathrm{NS}} V_2^{\mathrm{NS}} V_3^{\mathrm{NS}} V_4^{\mathrm{R\text{-}R}} \right\rangle , \tag{3.1}$$

where

$$\begin{aligned} V_1^{\mathrm{NS}}(k_1, \zeta_1, x_1) &= \zeta_{1\mu} : V_{-1}^{\mu}(2k_1, x_1) : \\ V_2^{\mathrm{NS}}(k_2, \zeta_2, x_2) &= \zeta_{2j} : V_0^{j}(2k_2, x_2) : \\ V_3^{\mathrm{NS}}(k_3, \zeta_3, x_3) &= \zeta_{3i} : V_0^{i}(2k_3, x_3) : \\ V_4^{\mathrm{R\text{-}R}}(p_4, c_4, z_4, \bar{z}_4) &= (P_- \Gamma_{4(n)} M_p)^{AB} : V_{-\frac{1}{2}A}(p_4, z_4) : : V_{-\frac{1}{2}B}(p_4 \cdot D, \bar{z}_4) : , \end{aligned} \tag{3.2}$$

and

$$\begin{aligned} V_0^{\mu}(p, z) &= (\partial X^{\mu} + ip \cdot \psi \psi^{\mu}) e^{ip \cdot X} , \\ V_{-1}^{\mu}(p, z) &= e^{-\sigma} \psi^{\mu} e^{ip \cdot X} , \\ V_{-\frac{1}{2}A}(p, z) &= e^{-\frac{1}{2}\sigma} S_A e^{ip \cdot X} . \end{aligned} \tag{3.3}$$

The vertex operators above are chosen such that they saturate the background superghost charge on the world-sheet, i.e. $Q_\sigma = 2$. In the first vertex operator, the index μ will run over the world-volume (transverse) directions when it represents a world-volume vector (transverse scalar) state. Here we are using the notation of ref. [15]. In particular, we have used the doubling trick [14, 15] to convert the disk amplitude to a calculation involving only the standard holomorphic correlators

$$\begin{aligned} \langle\, X^{\mu}(z)\, X^{\nu}(w)\, \rangle &= -\eta^{\mu\nu} \log(z - w) , \\ \langle\, \psi^{\mu}(z)\, \psi^{\nu}(w)\, \rangle &= -\frac{\eta^{\mu\nu}}{z - w} , \\ \langle\, \sigma(z)\, \sigma(w)\, \rangle &= -\log(z - w) . \end{aligned} \tag{3.4}$$

The necessary correlation functions between the world-sheet fermions and the spin operators appearing in (3.1) are

$$\langle : \psi^{\mu}(x_1) : : S_A(z_4) : : S_B(\bar{z}_4) : \rangle = \frac{1}{\sqrt{2}} (\gamma^{\mu})^{AB} x_{14}^{-1/2} x_{15}^{-1/2} x_{45}^{-3/4} , \tag{3.5}$$

and

$$: \psi^{\mu} \psi^{\nu}(x_2) : : S_A(z_4) : \simeq -\frac{1}{2} (\Gamma^{\mu\nu})_A{}^B S_B(z_4) x_{24}^{-1} , \tag{3.6}$$

where $\Gamma^{\mu\nu} = (\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu)/2$ and we have defined $x_4 \equiv z_4$, $x_5 \equiv \bar{z}_4$ and $x_{ij} = x_i - x_j$. Using the world-sheet fermion correlations (3.4) and (3.6), one can reduce all the correlators in (3.1) between the world-sheet fermions and the spin operators to the correlation function (3.5). After also performing the world-sheet bosonic correlation functions, the scattering amplitude in eq. (3.1) can be put in the following form:

$$
\begin{aligned}
&A_{123}(C^{(n-1)},A,2\Phi) = \\
&= \frac{\lambda^2\mu_p}{4\pi}\,\mathrm{Tr}(\zeta_{1a}\zeta_{2j}\zeta_{3i})(P_-\Gamma_{4(n)}M_p)^{AB}\int dx_1dx_2dx_3dx_4dx_5\, I\,\times \\
&\quad\times\Big[\eta^{ij}(1-4k_2\cdot k_3)(\gamma^a)_{AB}a_1+\big(p_4^ip_4^j(\gamma^a)_{AB}\;-p_4^j(\gamma^ik_3\!\cdot\!\gamma\gamma^a)_{AB}- \\
&\qquad\qquad -p_4^i(\gamma^jk_2\!\cdot\!\gamma\gamma^a)_{AB}+(k_2\!\cdot\!\gamma\gamma^j\gamma^i\gamma^ak_3\!\cdot\!\gamma)_{AB}\big)\,a_2+ \\
&\qquad+2k_3^ap_4^j(\gamma^i)_{AB}a_3+\big(2k_2^ap_4^i(\gamma^j)_{AB}+2k_2^a(k_3\!\cdot\!\gamma\gamma^i\gamma^j)_{AB}\big)\,a_4+ \\
&\qquad+2k_3^a(k_2\!\cdot\!\gamma\gamma^j\gamma^i)_{AB}a_5-\big(2k_2\cdot k_3(\gamma^i\gamma^j\gamma^a)_{AB}-2\eta^{ij}(k_2\!\cdot\!\gamma k_3\!\cdot\!\gamma\gamma^a)_{AB}\big)\,a_6- \\
&\qquad-4k_3^a\eta^{ij}(k_2\!\cdot\!\gamma)_{AB}a_7+4k_2^a\eta^{ij}(k_3\!\cdot\!\gamma)_{AB}a_8\Big]
\end{aligned}
\tag{3.7}
$$

$$
\begin{aligned}
&A_{123}(C^{(n-1)},3\Phi) = \\
&= \frac{\lambda^2\mu_p}{4\pi}\,\mathrm{Tr}(\zeta_{1i}\zeta_{2j}\zeta_{3k})(P_-\Gamma_{4(n)}M_p)^{AB}\int dx_1dx_2dx_3dx_4dx_5\, I\,\times \\
&\quad\times\Big[\big(p_4^jp_4^k(\gamma^i)_{AB}+p_4^k(k_2\!\cdot\!\gamma\gamma^j\gamma^i)_{AB}+p_4^j(k_3\!\cdot\!\gamma\gamma^k\gamma^i)_{AB}- \\
&\qquad-(k_3\!\cdot\!\gamma\gamma^k\gamma^i\gamma^jk_2\!\cdot\!\gamma)_{AB}\big)\,a_2-2\eta^{ij}p_4^k(k_2\!\cdot\!\gamma)_{AB}a_4+\big(2\eta^{ik}(k_3\!\cdot\!\gamma\gamma^jk_2\!\cdot\!\gamma)_{AB}- \\
&\qquad-2\eta^{ik}p_4^j(k_3\!\cdot\!\gamma)_{AB}\big)\,a_9-2\eta^{ij}(k_3\!\cdot\!\gamma\gamma^kk_2\!\cdot\!\gamma)_{AB}a_{10}+\big(2\eta^{jk}(k_3\!\cdot\!\gamma\gamma^ik_2\!\cdot\!\gamma)_{AB}- \\
&\qquad-2k_2\!\cdot\!k_3(\gamma^j\gamma^k\gamma^i)_{AB}\big)\,a_{11}+4k_2\!\cdot\!k_3\,\big(-\eta^{jk}(\gamma^i)_{AB}a_1+\eta^{ij}(\gamma^k)_{AB}a_8- \\
&\qquad\qquad-\eta^{ik}(\gamma^j)_{AB}a_{12}\big)+\eta^{jk}(\gamma^i)_{AB}a_1\Big]\,,
\end{aligned}
\tag{3.8}
$$

where

$$
\begin{aligned}
I &\equiv x_{12}^{4k_1\cdot k_2}x_{13}^{4k_1\cdot k_3}x_{14}^{2k_1\cdot p_4}x_{15}^{2k_1\cdot p_4}x_{32}^{4k_2\cdot k_3}x_{24}^{2k_2\cdot p_4}x_{25}^{2k_2\cdot p_4}x_{34}^{2k_3\cdot p_4}x_{35}^{2k_3\cdot p_4}x_{45}^{p_4\cdot D\cdot p_4}\,, \\
a_1 &\equiv x_{32}^{-2}(x_{14}x_{15}x_{45})^{-1}\,, \\
a_2 &\equiv x_{45}(x_{34}x_{35}x_{24}x_{25}x_{14}x_{15})^{-1}\,, \\
a_3 &\equiv (x_{14}x_{35}x_{13}x_{24}x_{25})^{-1}\,, \\
a_4 &\equiv (x_{14}x_{25}x_{12}x_{34}x_{35})^{-1}\,, \\
a_5 &\equiv (x_{15}x_{13}x_{34}x_{24}x_{25})^{-1}\,, \\
a_6 &\equiv (x_{14}x_{15}x_{34}x_{25}x_{32})^{-1}\,, \\
a_7 &\equiv (x_{14}x_{45}x_{13}x_{32}x_{25})^{-1}\,, \\
a_8 &\equiv (x_{14}x_{45}x_{35}x_{12}x_{32})^{-1}\,, \\
a_9 &\equiv (x_{13}x_{14}x_{24}x_{25}x_{35})^{-1}\,, \\
a_{10} &\equiv (x_{12}x_{24}x_{15}x_{34}x_{35})^{-1}\,, \\
a_{11} &\equiv (x_{14}x_{15}x_{32}x_{24}x_{35})^{-1}\,, \\
a_{12} &\equiv x_{34}(x_{13}x_{14}x_{32}x_{24}x_{35}x_{45})^{-1}\,.
\end{aligned}
\tag{3.9}
$$

Note that the gamma matrices appear inside traces. That is

$$(P_{-}\Gamma_{4(n)}M_p)^{AB}(\gamma^{\mu_1}\cdots\gamma^{\mu_n})_{AB} = \mathrm{tr}(P_{-}\Gamma_{4(n)}M_p\gamma^{\mu_n}\cdots\gamma^{\mu_1})\,, \tag{3.10}$$

where we use $\mathrm{tr}(\cdots)$ to denote the trace on gamma matrix indices (as opposed to $\mathrm{Tr}(\cdots)$ for the non-abelian gauge trace).

A highly non-trivial check of the results in eqs. (3.7) and (3.8) is that the integrals are $\mathrm{SL}(2,\mathbb{R})$ invariant. To remove the associated divergence and properly evaluate the amplitude, we fix: $x_4 = i$, $x_5 = -i$, $x_1 = R \to \infty$. With this choice, one finds

$$L_j \equiv \int dx_1 dx_2 dx_3 dx_4 dx_5 I a_j \longrightarrow \int_{-\infty}^{\infty} dx_2 \int_{x_2}^{\infty} dx_3 J_j\,, \tag{3.11}$$

where $j = 1, 2, \ldots, 12$ and

$$\begin{aligned} J_j = {} & (2i)^{p_4\cdot D\cdot p_4 + n^j_{45}}(x_2 - i)^{2k_2\cdot p_4 + n^j_{24}}(x_2 + i)^{2k_2\cdot p_4 + n^j_{25}} \times \\ & \times (x_3 - i)^{2k_3\cdot p_4 + n^j_{34}}(x_3 + i)^{2k_3\cdot p_4 + n^j_{35}}(x_3 - x_2)^{4k_2\cdot k_3 + n^j_{32}}\,, \end{aligned} \tag{3.12}$$

and integer n^j_{kl} above is defined to be the power of x_{kl} in a_j. For example $n^1_{32} = -2$, $n^1_{14} = n^1_{15} = n^1_{45} = -1$, $n^1_{24} = n^1_{25} = n^1_{34} = n^1_{35} = 0$. These integrals can be evaluated — see appendix A for details — and the result may be written in the following form:

$$\begin{aligned} L_j = {} & -(-i)^{2(t+s+u)}\Gamma(-n^j_{32} - n^j_{24} - n^j_{25} - n^j_{34} - n^j_{35} - 2 - 2t - 2s - 2u) \times \\ & \times \big(\exp[-i\pi(n^j_{24} + n^j_{34} + u)]\sin[\pi(n^j_{32} + n^j_{25} + n^j_{35} + t + s)] \times \\ & \times \frac{\Gamma(1 + n^j_{32} - 2u)\Gamma(-1 - n^j_{32} - n^j_{35} - s + u)\Gamma(2 + n^j_{32} + n^j_{25} + n^j_{35} + t + s)}{\Gamma(-n^j_{35} - s - u)\Gamma(-n^j_{24} - n^j_{34} - t - s - 2u)} \times \\ & \times {}_3F_2(-n^j_{34} - s - u, 1 + n^j_{32} - 2u, 2 + n^j_{32} + n^j_{25} + n^j_{35} + t + s; \\ & \qquad 2 + n^j_{32} + n^j_{35} + s - u, -n^j_{24} - n^j_{34} - t - s - 2u; 1) + \\ & + \exp[-i\pi(n^j_{32} + n^j_{24} + n^j_{34} + n^j_{35} + s)]\sin[\pi(n^j_{25} + t + u)] \times \\ & \times \frac{\Gamma(-1 - n^j_{32} - n^j_{34} - n^j_{35} - 2s)\Gamma(1 + n^j_{32} + n^j_{35} + s - u)\Gamma(1 + n^j_{25} + t + u)}{\Gamma(-n^j_{34} - s - u)\Gamma(-1 - n^j_{32} - n^j_{24} - n^j_{34} - n^j_{35} - t - 2s - u)} \times \\ & \times {}_3F_2(-n^j_{35} - s - u, -1 - n^j_{32} - n^j_{34} - n^j_{35} - 2s, 1 + n^j_{25} + t + u; \\ & \qquad - n^j_{32} - n^j_{35} - s + u, -1 - n^j_{32} - n^j_{24} - n^j_{34} - n^j_{35} - t - 2s - u; 1)\big)\,, \end{aligned} \tag{3.13}$$

where ${}_3F_2$ is a generalized hypergeometric function (see appendix A) and the three independent Mandelstam variables are defined in eq. (2.6). A careful examination of these results reveals that the amplitudes of interest potentially have poles at $s, t, u, s+t+u = 0, 1/2, 1, 3/2, 2, \ldots$ reflecting the infinite tower of open string states corresponding to excitation of the Dp-branes.

Now we are interested in the low energy limit: $s, t, u \to 0$.[1] Using the standard expansion for the gamma function, and the following expansion for the hypergeometric function [16]

$$
\begin{aligned}
{}_3F_2(a,b,c;d,e;1) = 1 - \frac{abc}{de(a+b+c-d-e)} \times \\
\times\Big(1 + [\zeta(2) + (a+b+c-d-e)\zeta(3)] \times \\
\times [(a+b+c-d-e)(b+c-d-e) - (b-d)(c-e+b) - \\
- c(c-e)] + \\
+ \zeta(3)[a(b-d)(c-e) - bd(b-d) - ce(c-e) + \\
+ (a+b+c-d-e)\times(-e(c-e) - d(b-d))] + \\
+ \cdots\Big),
\end{aligned}
$$

one finds that, in the low energy limit, eq. (3.13) yields:

$$
\begin{aligned}
L_1^{\rm low} &= -i\frac{\pi^2}{2}\left(u + \frac{st}{t+s+u}\right), \\
L_2^{\rm low} &= i\pi^2, \\
L_3^{\rm low} &= -\frac{\pi}{2}\left(\frac{1}{s}\right) - i\frac{\pi^2}{2}, \\
L_4^{\rm low} &= \frac{\pi}{2}\left(\frac{1}{t}\right) - i\frac{\pi^2}{2}, \\
L_5^{\rm low} &= -\frac{\pi}{2}\left(\frac{1}{s}\right) + i\frac{\pi^2}{2}, \\
L_6^{\rm low} &= -\frac{\pi}{2}\left(\frac{1}{u}\right) - i\frac{\pi^2}{2}, \\
L_7^{\rm low} &= \frac{\pi}{4}\left(\frac{1}{s} + \frac{1}{u}\right) + i\frac{\pi^2}{4}\left(1 - \frac{t}{t+s+u}\right), \\
L_8^{\rm low} &= \frac{\pi}{4}\left(\frac{1}{t} + \frac{1}{u}\right) - i\frac{\pi^2}{4}\left(1 - \frac{s}{t+s+u}\right), \\
L_{11}^{\rm low} &= -\frac{\pi}{2}\left(\frac{1}{u}\right) - i\frac{\pi^2}{2}.
\end{aligned}
\tag{3.14}
$$

We have not listed the results for $L_9^{\rm low}$, $L_{10}^{\rm low}$ and $L_{12}^{\rm low}$ as they are not needed in evaluating the amplitudes of interest in this section — see below.

Consider the amplitude (3.8) describing the scattering of the RR field with three world-volume scalars. For the case of interest, we have $n = p + 4$ and one finds that many of the terms in eq. (3.8) vanish. These vanishings result because after

[1] We are using conventions where α' is fixed, i.e. $\alpha' = 2$. Note that this convention fixes $\lambda = 4\pi$.

performing the trace over the gamma matrices (3.10), some of the indices of the RR field strength (implicit in $\Gamma_{4(n)}$) are contracted. The only non-zero terms are

$$\begin{aligned}
A_{123}(C^{(p+3)},3\Phi) \;&=\; -\frac{\lambda^2\mu_p}{4\pi}\,\mathrm{Tr}(\zeta_{1i}\zeta_{2j}\zeta_{3k})(P_-\Gamma_{4(p+4)}M_p)^{AB}\times\\
&\quad\times\int dx_1dx_2dx_3dx_4dx_5\, I\,\big((k_3\!\cdot\!\gamma\gamma^k\gamma^i\gamma^j k_2\!\cdot\!\gamma)_{AB}a_2+\\
&\qquad\qquad + 2k_2\!\cdot\! k_3(\gamma^j\gamma^k\gamma^i)_{AB}a_{11}\big) \qquad\qquad (3.15)\\
&\longrightarrow -\frac{\lambda^2\mu_p}{4\pi}\,\mathrm{Tr}(\zeta_{1i}\zeta_{2j}\zeta_{3k})\,\big(\mathrm{tr}(P_-\Gamma_{4(p+4)}M_pk_2\!\cdot\!\gamma\gamma^j\gamma^i\gamma^k k_3\!\cdot\!\gamma)L_2+\\
&\qquad\qquad + 2k_2\!\cdot\! k_3\,\mathrm{tr}(P_-\Gamma_{4(p+4)}M_p\gamma^i\gamma^k\gamma^j)L_{11}\big)\,.
\end{aligned}$$

Now substituting in $L_2^{\rm low}$ and $L_{11}^{\rm low}$, the leading order amplitude becomes

$$A_{123}^{\rm low}(C^{(p+3)},3\Phi) = -\frac{\lambda^2\mu_p}{8}\,\mathrm{Tr}(\zeta_{1i}\zeta_{2j}\zeta_{3k})\,\mathrm{tr}(P_-\Gamma_{4(p+4)}M_p\gamma^i\gamma^k\gamma^j)\,,$$

where we have used the fact that the $k_2\!\cdot\!\gamma$ and $k_3\!\cdot\!\gamma$ in the trace of gamma matrices in the first term of (3.15) will contract only with each other. Averaging two non-cyclic orderings of open string vertex operators and performing the trace over the gamma matrices then yields

$$A^{\rm low}(C^{(p+3)},3\Phi) = -\frac{\lambda^2\mu_p}{(p+1)!}\,\mathrm{Tr}(\zeta_{1i}[\zeta_{2j},\zeta_{3k}])(\varepsilon^v)_{a_0\cdots a_p}(\mathcal{F}_4^{(p+4)})^{ijka_0\cdots a_p},$$

where $\mathcal{F}_4^{(p+4)}$ denotes the linearized field strength. That is [15]

$$(\mathcal{F}_4^{(n)})^{\mu_1\cdots\mu_n} = in\,p_4^{[\mu_1}c_4^{\mu_2\cdots\mu_n]}\,,$$

where c_4 is the polarization tensor associated with the RR state. It clear that this leading term is precisely that arising from an interaction in the low energy action of the form given in eq. (2.2).

Now we turn to the case where the RR state scatters with two scalars and one gauge field. Performing the integrals in eq. (3.7) as described above and then substituting in the $L_j^{\rm low}$ given in eq. (3.14) yields

$$A_{123}^{\rm low}(C^{(n-1)},A,2\Phi) = A_{123}^c + \frac{A_{123}^t}{t} + \frac{A_{123}^s}{s} + \frac{A_{123}^u}{u} + \frac{A_{123}^{t+s+u}}{t+s+u}\,,$$

where

$$\begin{aligned}
A_{123}^c = &\frac{i\lambda^2\mu_p\pi}{4}\,\mathrm{Tr}(\zeta_{1a}\zeta_{2j}\zeta_{3i})(P_-\Gamma_{4(n)}M_p)^{AB}\times\\
&\times\big(k_2\cdot k_3\eta^{ij}(\gamma^a)_{AB} + p_4^ip_4^j(\gamma^a)_{AB} - p_4^j(\gamma^ik_3\!\cdot\!\gamma\gamma^a)_{AB} - p_4^i(\gamma^jk_2\!\cdot\!\gamma\gamma^a)_{AB} +\\
&\quad + (k_2\!\cdot\!\gamma\gamma^j\gamma^i\gamma^ak_3\!\cdot\!\gamma)_{AB} - k_3^ap_4^j(\gamma^i)_{AB} - k_2^ap_4^i(\gamma^j)_{AB} - k_2^a(k_3\!\cdot\!\gamma\gamma^i\gamma^j)_{AB} +
\end{aligned}$$

$$
\begin{aligned}
&\quad + k_3^a(k_2\cdot\gamma\gamma^j\gamma^i)_{AB} + k_2\cdot k_3(\gamma^i\gamma^j\gamma^a)_{AB} - \eta^{ij}(k_2\cdot\gamma k_3\cdot\gamma\gamma^a)_{AB} - \\
&\quad - k_3^a\eta^{ij}(k_2\cdot\gamma)_{AB} - k_2^a\eta^{ij}(k_3\cdot\gamma)_{AB}\big)\,, \\
A_{123}^t &= \frac{\lambda^2\mu_p}{4}\,\mathrm{Tr}(\zeta_1\cdot k_2\zeta_{2j}\zeta_{3i})(P_-\Gamma_{4(n)}M_p)^{AB}\,\big(p_4^i(\gamma^j)_{AB} + (k_3\cdot\gamma\gamma^i\gamma^j)_{AB} + \\
&\qquad + \eta^{ij}(k_3\cdot\gamma)_{AB}\big)\,, \\
A_{123}^s &= -\frac{\lambda^2\mu_p}{4}\,\mathrm{Tr}(\zeta_1\cdot k_3\zeta_{2j}\zeta_{3i})(P_-\Gamma_{4(n)}M_p)^{AB}\,\big(p_4^j(\gamma^i)_{AB} + (k_2\cdot\gamma\gamma^j\gamma^i)_{AB} + \\
&\qquad + \eta^{ij}(k_2\cdot\gamma)_{AB}\big)\,, \\
A_{123}^u &= \frac{\lambda^2\mu_p}{4}\,\mathrm{Tr}(\zeta_{1a}\zeta_{2j}\zeta_{3i})(P_-\Gamma_{4(n)}M_p)^{AB}\,\big(k_2\cdot k_3(\gamma^i\gamma^j\gamma^a)_{AB} - \eta^{ij}(k_2\cdot\gamma k_3\cdot\gamma\gamma^a)_{AB} - \\
&\qquad - k_3^a\eta^{ij}(k_2\cdot\gamma)_{AB} + k_2^a\eta^{ij}(k_3\cdot\gamma)_{AB}\big)\,, \\
A_{123}^{t+s+u} &= \frac{i\lambda^2\mu_p\pi}{4}\,\mathrm{Tr}(\zeta_{1a}\zeta_2\cdot\zeta_3)(P_-\Gamma_{4(n)}M_p)^{AB}\left(-\frac{1}{2}ts(\gamma^a)_{AB} + tk_3^a(k_2\cdot\gamma)_{AB} + \right. \\
&\qquad \left. + sk_2^a(k_3\cdot\gamma)_{AB}\right).
\end{aligned} \tag{3.16}
$$

This result from eq. (3.7) corresponds to just one ordering of open string states. Of course, the full scattering amplitude includes a sum over all non-cyclic permutations, and so the full low energy scattering amplitude has the form

$$
\begin{aligned}
A^{\mathrm{low}} &= \frac{1}{2}\left(A_{123}^c + A_{132}^c + \frac{A_{123}^t + A_{132}^s}{t} + \frac{A_{123}^s + A_{132}^t}{s} + \frac{A_{123}^u + A_{132}^u}{u} + \right. \\
&\qquad \left. + \frac{A_{123}^{s+t+u} + A_{132}^{s+t+u}}{s+t+u}\right) \\
&\equiv A^c + \frac{A^t}{t} + \frac{A^s}{s} + \frac{A^u}{u} + \frac{A^{s+t+u}}{s+t+u}\,.
\end{aligned} \tag{3.17}
$$

For the case of interest, $n = p+2$, i.e. we are considering $C^{(p+1)}$ coupling to a Dp-brane. After performing the gamma matrice traces in eq. (3.16), the coefficients in eq. (3.17) can be written as

$$
\begin{aligned}
A^c &= 0\,, \\
A^u &= -\frac{\lambda^2\mu_p}{p!}\,u\,\left(\mathrm{Tr}(\zeta_{1a_0}\zeta_2^j\zeta_3^i) + \mathrm{Tr}(\zeta_{1a_0}\zeta_3^j\zeta_2^i)\right)(\mathcal{F}_4^{(p+2)})_{ija_1\cdots a_p}(\varepsilon^v)^{a_0\cdots a_p}\,, \\
A^t &= \frac{2\lambda^2\mu_p}{(p+1)!}\Big(\left(\mathrm{Tr}(\zeta_1\cdot k_2\,\zeta_2^i\,\zeta_3\cdot p_4) - \mathrm{Tr}(\zeta_1\cdot k_2\,\zeta_3\cdot p_4\,\zeta_2^i)\right)(\mathcal{F}_4^{(p+2)})_{ia_0\cdots a_p} - \\
&\qquad - (p+1)\,\mathrm{Tr}\left(\zeta_1\cdot k_2\,[\zeta_2^i,\zeta_3^j]\right)k_{3a_0}(\mathcal{F}_4^{(p+2)})_{ija_1\cdots a_p}\Big)(\varepsilon^v)^{a_0\cdots a_p}, \\
A^s &= A^t(2\leftrightarrow 3)\,, \\
A^{s+t+u} &= 0\,.
\end{aligned}
$$

Since A^u above is proportional to u, the amplitude has no massless u-channel pole. Hence in agreement with the field theory analysis in the previous section, the only poles are in the s- and t-channels. In fact, we can match these poles in eq. (3.17) precisely with those arising in the amplitude calculated in the low energy world-volume field theory. In the field theory, the t-channel amplitude can be written as

$$A_t^{C_4\Phi_3\Phi_2A_1} = (\tilde{V}^{C_4\Phi_3\Phi})^i_\alpha (\tilde{G}^\Phi)^{ij}_{\alpha\beta} (\tilde{V}^{\Phi\Phi_2A_1})^j_\beta \,. \tag{3.18}$$

The vertices and propagator above, which can be read from eqs. (2.3) and (2.5), are

$$\begin{aligned}
(\tilde{V}^{C_4\Phi_3\Phi})^i_\alpha &= -\frac{N\lambda^2\mu_p}{(p+1)!}\left[(\zeta_{3\alpha}\!\cdot\! p_4)\,(\mathcal{F}_4^{(p+2)})^i{}_{a_0\cdots a_p} + (p+1)k_{3a_0}\zeta^j_{3\alpha}(\mathcal{F}_4^{(p+2)})^i{}_{ja_1\cdots a_p}\right]\times\\
&\quad\times(\varepsilon^v)^{a_0\cdots a_p}\,,\\
(\tilde{V}^{\Phi\Phi_2A_1})^j_\beta &= -2i\lambda^2T_p\,\mathrm{Tr}\left(\zeta_1\!\cdot\! k_2[\zeta^j_2, T_\beta]\right),\\
(\tilde{G}^\Phi)^{ij}_{\alpha\beta} &= -\frac{i}{N\lambda^2T_p}\frac{\delta^{ij}\delta_{\alpha\beta}}{q^2}\,,
\end{aligned} \tag{3.19}$$

where $q = k_1 + k_2$. We have also written $\zeta^i = \zeta^i_\alpha T_\alpha$ where T_α are the $\mathrm{U}(N)$ generators with normalization $\mathrm{Tr}(T_\alpha T_\beta) = N\delta_{\alpha\beta}$. We have simplified the vertex in the first line above using the Bianchi identity $dF^{(p+2)} = 0$, which yields

$$0 = \left[(p+1)p_{4a_0}(\mathcal{F}_4^{(p+2)})_{ija_1\cdots a_p} + p_{4i}(\mathcal{F}_4^{(p+2)})_{ja_0\cdots a_p} - p_{4j}(\mathcal{F}_4^{(p+2)})_{ia_0\cdots a_p})\right](\varepsilon^v)^{a_0\cdots a_p}\,.$$

Substituting eq. (3.19) into eq. (3.18), one finds

$$A_t^{C_4\Phi_3\Phi_2A_1} = \frac{(A^t)}{t}\,,$$

and so there is precise agreement between the field theory calculation and the t-channel pole in the string amplitude. A similar calculation in s-channel also yields agreement,

$$A_s^{C_4\Phi_2\Phi_3A_1} = \frac{(A^s)}{s}\,.$$

Finally it is straightforward to show that the contact term A^u/u exactly reproduces the interaction in the last line of eq. (2.5).

4. Born-Infeld string amplitudes

Finally we consider the string scattering amplitude of one closed string NS-NS state and three open string scalars on a Dp-brane:

$$A = \frac{\lambda^2T_p}{\pi}\,\mathrm{Tr}\int dx_1dx_2dx_3d^2z_4\,\left\langle V_1^{\mathrm{NS}}\,V_2^{\mathrm{NS}}\,V_3^{\mathrm{NS}}\,V_4^{\mathrm{NS\text{-}NS}}\right\rangle, \tag{4.1}$$

where the vertex operators are

$$\begin{aligned}
V_\ell^{\mathrm{NS}}(k_\ell,\zeta_\ell,x_\ell) &= \zeta_{\ell i} : V_0^i(2k_\ell, x_\ell) : \qquad \ell = 1,2,3\,,\\
V_4^{\mathrm{NS\text{-}NS}}(p_4,\varepsilon_4,z_4,\bar{z}_4) &= (\varepsilon_4\!\cdot\! D)_{\mu\nu}\; : V^\mu_{-1}(p_4,z_4) : : V^\nu_{-1}(p_4\!\cdot\! D,\bar{z}_4) :,
\end{aligned}$$

with V_0 and V_{-1} given in eq. (3.3). Using the propagators given in eq. (3.4), one can calculate the correlators to produce a rather lengthy result of the same basic form as in eqs. (3.7) and (3.8) where the various kinematic factors come with one or three momenta. In an effort to reduce the calculations (and the presentation), consider that we are interested in the particular set of low energy interactions given in eq. (2.7). Hence we must identify the contact terms in the amplitude which have only one momentum. These can be produced in two different ways: if the kinematic factor has one momentum, the corresponding integrals may yield a constant term in the low energy limit. Alternatively, in terms with three momenta, two momenta may be contracted to yield a factor of s, t or u while the corresponding integral produces a massless pole in the same channel. Thus one would again be left with contact terms containing a single momentum.

Let us begin with the first case. The relevant contributions are

$$\begin{aligned} A^{(1)} &= \frac{i\lambda^2 T_p}{\pi}\,\mathrm{Tr}(\zeta_{1i}\zeta_{2j}\zeta_{3k})\,(\varepsilon_4\!\cdot\! D)_{\mu\nu}\int dx_1dx_2dx_3dx_4dx_5\, I\,\times \\ &\times\Big(\Big(\eta^{\mu\nu}\eta^{jk}p_4^i+4\eta^{jk}\eta^{i[\mu}k_1^{\nu]}\Big)\,a_1+\Big(\eta^{\mu\nu}\eta^{ij}p_4^k+4\eta^{ij}\eta^{k[\mu}k_3^{\nu]}\Big)\,a_{13}\,+ \\ &\qquad +\Big(\eta^{\mu\nu}\eta^{ik}p_4^j+4\eta^{ik}\eta^{j[\mu}k_2^{\nu]}\Big)\,a_{14}\Big)\,, \end{aligned} \tag{4.2}$$

where a_1 is defined in eq. (3.9), while a_{13} and a_{14} are

$$\begin{aligned} a_{13} &\equiv x_{12}^{-2}(x_{34}x_{35}x_{45})^{-1}\,, \\ a_{14} &\equiv x_{13}^{-2}(x_{24}x_{25}x_{45})^{-1}\,. \end{aligned} \tag{4.3}$$

Now one finds that in the low energy limit, the corresponding integrals (3.13) yield no constant part in $L_1^{\rm low}$ (see eq. (3.14)), $L_{13}^{\rm low}$ or $L_{14}^{\rm low}$. Hence, this contribution (4.2) to the amplitude gives no contact terms of the desired form, i.e. with one momentum.

Now we turn to the second case. The terms of interest are

$$\begin{aligned} A^{(3)} &= -\frac{2i\lambda^2 T_p}{\pi}\,\mathrm{Tr}(\zeta_{1i}\zeta_{2j}\zeta_{3k})\,(\varepsilon_4\!\cdot\! D)_{\mu\nu}\int dx_1dx_2dx_3dx_4dx_5\, I\,\times \\ &\times\Big(t\,\Big[(\eta^{\mu\nu}\eta^{ij}p_4^k+4\eta^{ij}\eta^{k[\mu}k_3^{\nu]}\Big)\,a_{13}+\eta^{i\nu}\eta^{j\mu}p_4^ka_{11}-\eta^{i\mu}\eta^{j\nu}p_4^ka_4+ \\ &\qquad +2\eta^{ik}\eta^{j\nu}k_3^\mu a_{15}+2\eta^{jk}\eta^{i\nu}k_3^\mu a_{16}-2\eta^{ik}\eta^{j\mu}k_3^\nu a_{17}-2\eta^{jk}\eta^{i\mu}k_3^\nu a_8\Big]\,+ \\ &\quad +\ s\,\Big[\Big(\eta^{\mu\nu}\eta^{ik}p_4^j+4\eta^{ik}\eta^{j[\mu}k_2^{\nu]}\Big)\,a_{14}+\eta^{i\nu}\eta^{k\mu}p_4^ja_5-\eta^{k\nu}\eta^{i\mu}p_4^ja_9+ \\ &\qquad +2\eta^{ij}\eta^{k\nu}k_2^\mu a_{17}-2\eta^{jk}\eta^{i\nu}k_2^\mu a_{18}-2\eta^{ij}\eta^{k\mu}k_2^\nu a_{15}+2\eta^{jk}\eta^{i\mu}k_2^\nu a_{19}\Big]\,+ \\ &\quad +\ u\,\Big[\Big(\eta^{\mu\nu}\eta^{jk}p_4^i+4\eta^{jk}\eta^{i[\mu}k_1^{\nu]}\Big)\,a_1+\eta^{j\mu}\eta k\nu p_4^ia_{11}-\eta^{j\nu}\eta^{k\mu}p_4^ia_6+ \\ &\qquad +2\eta^{ij}\eta^{k\nu}k_1^\mu a_8-2\eta^{ik}\eta^{j\nu}k_1^\mu a_{19}-2\eta^{ij}\eta^{k\mu}k_1^\nu a_{16}+2\eta^{ik}\eta^{j\mu}k_1^\nu a_{18}\Big]\Big)\,. \end{aligned}$$

Beyond the a_i given in eqs. (3.9) and (4.3), we have defined

$$\begin{aligned}
a_{15} &\equiv (x_{12}x_{13}x_{25}x_{34}x_{45})^{-1}\,,\\
a_{16} &\equiv (x_{12}x_{32}x_{15}x_{34}x_{45})^{-1}\,,\\
a_{17} &\equiv (x_{12}x_{13}x_{24}x_{35}x_{45})^{-1}\,,\\
a_{18} &\equiv (x_{13}x_{15}x_{32}x_{24}x_{45})^{-1}\,,\\
a_{19} &\equiv (x_{13}x_{14}x_{32}x_{25}x_{45})^{-1}\,.
\end{aligned}$$

Evaluating the integrals as in eq. (3.13), one can extract the massless poles in each. As well as those given in eq. (3.14), we need

$$\begin{aligned}
L_9^{\rm low} &= -\frac{\pi}{2}\left(\frac{1}{s}\right),\\
L_{10}^{\rm low} &= \frac{\pi}{2}\left(\frac{1}{t}\right),\\
L_{15}^{\rm low} &= \frac{\pi}{4}\left(\frac{1}{t}+\frac{1}{s}\right),\\
L_{16}^{\rm low} &= -\frac{\pi}{4}\left(\frac{1}{t}+\frac{1}{u}\right),\\
L_{17}^{\rm low} &= -\frac{\pi}{4}\left(\frac{1}{t}+\frac{1}{s}\right),\\
L_{18}^{\rm low} &= -\frac{\pi}{4}\left(\frac{1}{s}+\frac{1}{u}\right),\\
L_{19}^{\rm low} &= \frac{\pi}{4}\left(\frac{1}{s}+\frac{1}{u}\right).
\end{aligned}$$

Further $L_{13}^{\rm low}$ and $L_{14}^{\rm low}$ appear in $A^{(3)}$, but they do not have any massless poles.

With the above results, one finds the following contact terms with only one momentum for the NS 2-form

$$\begin{aligned}
A^c_{123}(B,3\Phi) = -2i\lambda^2 T_p\,\big(&{\rm Tr}(\zeta_1\!\cdot\!\varepsilon_4\!\cdot\!\zeta_2\,p_4\!\cdot\!\zeta_3) + {\rm Tr}(\zeta_2\!\cdot\!\varepsilon_4\!\cdot\!k_3\,\zeta_3\!\cdot\!\zeta_1) -\\
&- {\rm Tr}(\zeta_3\!\cdot\!\varepsilon_4\!\cdot\!k_2\,\zeta_1\!\cdot\!\zeta_2) + \text{cyclic permutations of (123)}\big)\,.
\end{aligned}$$

This result corresponds to one ordering of the external open string states. Adding the two non-cyclic permutations of these states yields

$$\begin{aligned}
A^c(B,3\Phi) &= \frac{1}{2}\left(A^c_{123}(B,3\Phi) + A^c_{132}(B,3\Phi)\right)\\
&= -i\lambda^2 T_p\Bigg(\frac{1}{3}\,{\rm Tr}(\zeta_1^i\zeta_2^j\zeta_3^k + \zeta_2^i\zeta_1^j\zeta_3^k)\,(p_{4i}\varepsilon_{4jk} + p_{4j}\varepsilon_{4ki} + p_{4k}\varepsilon_{4ij}) +\\
&\qquad + {\rm Tr}[\zeta_2\!\cdot\!\varepsilon_4\!\cdot\!k_3\,(\zeta_3\!\cdot\!\zeta_1 - \zeta_1\!\cdot\!\zeta_3)] + \text{cyclic permutations of (123)}\Bigg)\,.
\end{aligned}$$

It is not difficult to verify that these contributions to the amplitude are reproduced by the interactions in eq. (2.7).

Further one can show that the corresponding contact terms vanish if one chooses to evaluate the amplitude for either a graviton or dilaton polarization tensor. Again this is in agreement with the low energy field theory where no interactions with a single derivative were found for these fields. Finally one would like to verify that the string theory amplitude has no massless poles in accord with the field theory analysis. While calculating the entire string amplitude would be a very lengthy task, a simple way to verify the absence of any poles is to calculate the amplitude (4.1) for $k_1 = k_2 = k_3 = p_4 = 0$. In this case, it is easy to check that the amplitude is zero, so the whole amplitude has no massless poles.

5. Discussion

In the non-abelian world-volume theory of N coincident Dp-branes, the transverse scalars transform in the adjoint of the U(N) gauge symmetry. Hence one has the possibility of new interactions involving commutators $[\Phi^i, \Phi^j]$ which could not appear in the abelian theory describing a single D-brane. The simplest example, of course, is that at leading order in the low energy expansion, the scalars have a non-trivial potential which is the square of two such commutators. The latter [8] arises from the expansion of the $\det(Q)$ factor in the non-abelian Born-Infeld action (1.1). Similarly, the interactions with the bulk supergravity fields are modified by the appearance of commutator terms. These appear both in the Born-Infeld term (1.1), through the contributions involving the matrix Q, and in the Wess-Zumino term (1.3), from the exponential of $\mathrm{i}_\Phi \mathrm{i}_\Phi$. In refs. [8, 9], the existence of the commutator terms in the non-abelian action was deduced by demanding that the non-abelian theory must be consistent with T-duality and that it match the well-known abelian action.

In the present paper, we have confirmed the existence of a certain class of these new couplings by the direct examination of string scattering amplitudes. In section 4, we extracted contact terms that correspond to the leading order commutator interactions arising from Q in the Born-Infeld action. Note that at the order studied here, there are already contributions from both of the determinants appearing in eq. (1.1). The first term in eq. (2.7) originates in the factor of $\det(Q)$, while the second term is a contribution from the Q^{-1} in the first determinant factor. In section 3, we have extracted interactions in the Wess-Zumino action involving the first non-trivial contribution from the exponential factor, $\exp[i\lambda\, \mathrm{i}_\Phi \mathrm{i}_\Phi] \simeq 1 + i\lambda\, \mathrm{i}_\Phi \mathrm{i}_\Phi + \cdots$. These interactions are given in eqs. (2.1) and (2.4). Note that beyond the commutator interactions, the calculations in section 3 also give direct evidence of the appearance of gauge covariant derivatives in the pull-back expressions [11, 12]. That is, in matching the contact terms in the amplitude involving two scalars and one gauge field, contributions involving the gauge field commutator in $D_a \Phi^i$ were essential in producing the final result in eq. (2.5).

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These new commutator interactions further enrich the diverse array of interesting physical properties displayed by Dp-branes. In particular, in the Wess-Zumino term (1.3), interactions appear involving the RR potentials with a higher form degree. Hence in the non-abelian theory, a Dp-brane can also couple to the RR potentials $C^{(n)}$ with $n = p+3, p+5, \ldots$ through the additional commutator interactions. Of course, these interactions are reminiscent of those discussed in matrix theory [17]. For example, the D0-brane action includes a linear coupling to $C^{(3)}$, the potential corresponding to D2-brane charge,

$$i\lambda\,\mu_0 \int \mathrm{Tr}\ P\left[\mathrm{i}_\Phi \mathrm{i}_\Phi C^{(3)}\right] = i\frac{\lambda}{2}\mu_0 \int dt\ \ \mathrm{Tr}\left(C^{(3)}_{tjk}(\Phi,t)\,[\Phi^k,\Phi^j] + \right.$$
$$\left. + \,\lambda C^{(3)}_{ijk}(\Phi,t)\,D_t\Phi^k\,[\Phi^k,\Phi^j]\right)\,. \quad (5.1)$$

The first term on the right-hand side has the form of a source for D2-brane charge, and is essentially the interaction central to the construction of M2-branes in matrix theory with the large-N limit [17]. However, with finite N, this term would vanish upon taking the trace if $C^{(3)}_{tjk}$ was simply a function of the world-volume coordinate t since $[\Phi^k, \Phi^j] \in \mathrm{SU}(N)$. Here though, eq. (5.1) yields non-trivial interactions since the three-form components are functionals of Φ^i. Hence, while there would be no "monopole" coupling to D2-brane charge, non-trivial expectation values of the scalars can give rise to couplings to an infinite series of higher "multipole" moments.

As well as allowing the Dp-branes to act as a source for higher RR fields, these new couplings also force the Dp-branes to respond to a non-trivial background RR field for which the branes would normally be regarded as neutral. That is, with non-trivial background fields, the commutator couplings induce new terms in the scalar potential, and hence generically one can expect that new extrema will be generated for the latter. In particular, there may be non-trivial extrema with non-commuting expectation values of the Φ^i, e.g. with $\mathrm{Tr}\,\Phi^i = 0$ but $\mathrm{Tr}(\Phi^i)^2 \neq 0$. This physical response corresponds to the external field "polarizing" the Dp-branes to expand into a non-commutative world-volume geometry [18]. Known as the "dielectric effect" [8], it is a direct analog of the dielectric effect in ordinary electromagnetism. This effect was first illustrated in ref. [8] with a simple toy calculation involving D0-branes in a background four-form field strength $F^{(4)}$. The interaction in eq. (2.2), whose presence was confirmed by the current calculations, was the essential coupling driving the dielectric effect in that example. It was also noted there that the D0-branes would respond to the NS three-form H in the same way because of the first interaction appearing eq. (2.7).

The dielectric effect has been found to play a role in a number of string theory contexts. For example, the resolution of certain singularities in the AdS/CFT correspondence has been explained in terms of external RR and/or NS fields polarizing D3-branes [19] — see also [20]. The analogous result has also been discussed in an M-theory framework [21]. The dielectric effect is also important in discussing the stabilization of D-branes in the spacetime background corresponding to a WZW model [22], as well as $AdS_m \times S^n$ backgrounds involving RR fields. Further, one can consider more sophisticated background field configurations which through the dielectric effect generate more complicated non-commutative geometries [24]. The most serious short-coming for the toy calculations in ref. [8] is that the background fields were not a consistent solution of the low energy equations of motion. One can find solutions with a constant background $F^{(4)}$ in M-theory, namely the $AdS_4 \times S^7$ and $AdS_7 \times S^4$ backgrounds — see, e.g. [25]. In lifting D0-branes to M-theory, they become gravitons carrying momentum in the internal space. Hence the expanded D2-D0 system of ref. [8] is related to the "giant gravitons" considered in refs. [26, 27, 28]. The analog of the D2-D0 bound state in a constant background $F^{(4)}$ corresponds to M2-branes with internal momentum expanding into AdS_4 [27], while that in a constant H field corresponds to the M2-branes expanding on S^4 [26].

It was noted in ref. [8] that the new potential terms which come into play for the dielectric effect only depend on the RR field strength, which should be expected for the results to be invariant under the RR gauge symmetry. Recall that in the string scattering amplitudes in section 3, one starts with a vertex operator written in terms of the RR field strength. Hence the resulting contact terms are naturally derived in terms of this field strength, and as a result are invariant under the RR gauge transformations. However, as presented in the Wess-Zumino action (1.3), the interactions are naturally written in terms of the RR potentials, and thus the RR gauge invariance is no longer manifest. Of course, for the interactions studied here, we have shown that the two representations of the RR couplings agree up to total derivatives. As seen in eq. (2.1) or eqs. (2.4)–(2.5), the necessary integration by parts requires what appears to be a complicated interplay between terms which have completely different origins (e.g. the interior products or the Taylor expansion or the pull-back) in the expansion of the Wess-Zumino action. A similar discussion applies for the NS couplings in the Born-Infeld action (1.1). In fact, it must be true that all of the world-volume interactions respect the appropriate spacetime gauge symmetries. However, given the action in eqs. (1.1) and (1.3), it remains an exercise to confirm these invariances on a case by case basis. Hence from this point of view, it seems that the description of the world-volume dynamics of D-branes is still lacking at some fundamental level.

In this paper, we have focussed our attention on limited set of interactions in the non-abelian world-volume action. Of course, one could extend our calculations to make a more extensive survey of the interactions appearing in the low energy action, and confirm in more detail the form of the non-abelian action given in eqs. (1.1) and (1.3). While we restrain ourselves from a complete analysis here, we will consider one additional example below. That is, we examine the scattering amplitude involving one gauge field, two transverse scalars and the RR $(p-1)$-form potential on a Dp-brane. This requires only a minor extension of the calculations already presented in section 3, namely, we set $n = p$ in eq. (3.7). The result provides evidence in favor of the use of the symmetrized trace in the Wess-Zumino action (1.3).

If one sets $n = p$ in eq. (3.7), the string scattering amplitude takes the form given in eq. (3.17) with

$$\begin{aligned} A^c &= -\frac{i\lambda^3\mu_p}{2p!}\,\mathrm{Tr}(\zeta_{1a_0}\zeta_2^j\zeta_3^i)\,\Big(p_{4i}p_{4j}(\mathcal{F}_4^{(p)})_{a_1\cdots a_p} + p\,k_{3a_1}p_{4j}(\mathcal{F}_4^{(p)})_{ia_2\cdots a_p} + \\ &\qquad + p\,k_{2a_1}p_{4i}(\mathcal{F}_4^{(p)})_{ja_2\cdots a_p} + \\ &\qquad + p(p-1)\,k_{3a_1}k_{2a_2}(\mathcal{F}_4^{(p)})_{ija_3\cdots a_p}\Big)\,(\varepsilon^v)^{a_0\cdots a_p} + \\ &\quad + [2\leftrightarrow 3]\,, \\ A^u &= A^t = A^s = 0\,, \\ A^{s+t+u} &= -\frac{i\lambda^3\mu_p}{2p!}\Big(-\frac{1}{2}st\ \mathrm{Tr}(\zeta_{1a_0}\zeta_2\cdot\zeta_3) + t\ \mathrm{Tr}(\zeta_1\cdot k_3\,\zeta_2\cdot\zeta_3)k_{2a_0} + \\ &\qquad + s\ \mathrm{Tr}(\zeta_1\cdot k_2\,\zeta_2\cdot\zeta_3)k_{3a_0}\Big)(\mathcal{F}_4^{(p)})_{a_1\cdots a_p}(\varepsilon^v)^{a_0\cdots a_p} + [2\leftrightarrow 3]\,. \end{aligned} \tag{5.2}$$

One can easily verify that the contact terms in A^c can be reproduced by the field interactions

$$\begin{aligned} S^{(vi)} &= \lambda\mu_p\int \mathrm{STr}\left(P\left[C^{(p-1)}(\sigma,\Phi)\right]F\right) \\ &= \frac{\lambda^3\mu_p}{4(p-1)!}\int d^{p+1}\sigma\,(\varepsilon^v)^{a_0\cdots a_p}\,\times \\ &\quad\times\Big(\mathrm{STr}(F_{a_0a_1}\Phi^i\Phi^j)\,\partial_i\partial_j C^{(p-1)}_{a_2\cdots a_p} + 2(p-1)\ \mathrm{STr}(F_{a_0a_1}\partial_{a_2}\Phi^i\Phi^j)\,\partial_j C^{(p-1)}_{ia_3\cdots a_p} + \\ &\qquad + (p-2)(p-1)\ \mathrm{STr}(F_{a_0a_1}\partial_{a_2}\Phi^i\partial_{a_3}\Phi^j)\,C^{(p-1)}_{ija_4\cdots a_p}\Big)\,, \\ &= \frac{\lambda^3\mu_p}{2p!}\int d^{p+1}\sigma\,(\varepsilon^v)^{a_0\cdots a_p}\,\times \\ &\quad\times\Big(\mathrm{STr}(A_{a_0}\Phi^i\Phi^j)\,\partial_i\partial_j F^{(p)}_{a_1\cdots a_p} + 2p\ \mathrm{STr}(A_{a_0}\partial_{a_1}\Phi^j\Phi^i)\,\partial_i F^{(p)}_{ja_2\cdots a_p} + \\ &\qquad + (p-1)p\ \mathrm{STr}(A_{a_0}\partial_{a_1}\Phi^i\partial_{a_2}\Phi^j)\,F^{(p)}_{ija_3\cdots a_p}\Big)\,, \end{aligned} \tag{5.3}$$

where the symmetric trace averages over all orderings of the three fields enclosed in each term [8]. In particular, in the second term of the final expression, one has

$$\mathrm{STr}(A_{a_0}\partial_{a_1}\Phi^j\Phi^i) \equiv \frac{1}{2}\,\mathrm{Tr}(A_{a_0}\partial_{a_1}\Phi^j\Phi^i + A_{a_0}\Phi^i\partial_{a_1}\Phi^j)\,. \tag{5.4}$$

For the first and third terms in the final result in eq. (5.3), the average over non-cyclic permutations is trivial because of the index symmetries of the fields in these interactions. Note that the symmetric averaging in the trace was essential in producing interactions which only involve $F^{(p)}$ in the final expression.

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It is straightforward to verify that the field theory will not produce any massless poles in the s-, t- or u-channels, in agreement with the vanishing of A^s, A^t and A^u in the string amplitude. However, since A^{s+t+u} is non-vanishing in eq. (5.2), the amplitude has a pole of the form $1/(p_4^\perp)^2 = 1/(s+t+u)$. Such a contribution arises in the low energy field theory if there was an interaction involving the RR form and a single world-volume field. For the RR $(p-1)$-form potential coupling to a Dp-form, the relevant interaction may be written as

$$S^{(vii)} = \frac{\lambda\mu_p}{p!}\int d^{p+1}\sigma\,(\varepsilon^v)^{a_0\cdots a_p}\ \mathrm{Tr}\left(A_{a_0}\right)\ F^{(p)}_{a_1\cdots a_p}(\sigma)\,, \tag{5.5}$$

which, of course, only involves the U(1) component of the gauge field. Now field theory amplitude in $(s+t+u)$-channel is given by

$$A^{C_4\Phi_3\Phi_2A_1}_{s+t+u} = (\tilde{V}^{C_4A})^a_\alpha\,(\tilde{G}^A)_{ab,\alpha\beta}\,(\tilde{V}^{A\Phi_3\Phi_2A_1})^b_\beta\,, \tag{5.6}$$

where the propagator is derived from the standard gauge kinetic term arising in the expansion of the Born-Infeld term:

$$(\tilde{G}^A)_{ab,\alpha\beta} = -\frac{i}{N\lambda^2T_p}\frac{\eta_{ab}\delta_{\alpha\beta}}{q^2}\,,$$

where $q = k_1+k_2+k_3$, and the first vertex is derived from the interaction given above in eq. (5.5):

$$(\tilde{V}^{C_4A})^a_\alpha = \frac{i\lambda\mu_p}{p!}(\mathcal{F}_4^{(p)})_{a_1\cdots a_p}\varepsilon^{aa_1\cdots a_p}\,\mathrm{Tr}(T_\alpha)\,.$$

The last vertex comes from an order λ^4 interaction in the Born-Infeld action

$$-\frac{\lambda^4T_p}{2}\left(\mathrm{STr}(D_a\Phi_iD_b\Phi^iF^{ac}F^b{}_c) - \frac{1}{4}\mathrm{STr}(D_a\Phi_iD^a\Phi^iF^{bc}F_{bc})\right),$$

and can be written as

$$\begin{aligned}(\tilde{V}^{A_1\Phi_2\Phi_3A})^b_\beta = \frac{i\lambda^4T_p}{2}\Bigg(&-\frac{1}{2}st\,\mathrm{Tr}(\zeta_2\cdot\zeta_3\zeta_1^bT_\beta) + sk_3^b\,\mathrm{Tr}(\zeta_2\cdot\zeta_3\zeta_1\cdot k_2T_\beta)+\\ &+tk_2^b\,\mathrm{Tr}(\zeta_2\cdot\zeta_3\zeta_1\cdot k_3T_\beta) + \frac{1}{2}sq^b\,\mathrm{Tr}(\zeta_2\cdot\zeta_3\zeta_1\cdot k_2T_\beta)+\\ &+\frac{1}{2}tq^b\,\mathrm{Tr}(\zeta_2\cdot\zeta_3\zeta_1\cdot k_3T_\beta)\Bigg) + [2\leftrightarrow 3]\,,\end{aligned}$$

where we used the fact that the off-shell gauge field must be abelian. The latter implies that $\mathrm{STr}(\cdots)$ is equivalent to $\mathrm{Tr}(\cdots)$ in this term. Replacing these vertices and propagator into (5.6), one reproduces the corresponding contribution in the string amplitude,

$$A^{C_4\Phi_3\Phi_2A_1}_{s+t+u} = \frac{A^{s+t+u}}{s+t+u}\,.$$

As noted in eq. (5.4) implementing the symmetric trace has a non-trivial effect on these calculations. In particular, having a symmetric trace is essential in producing the expressions in eq. (5.3) which are invariant under the RR gauge symmetry. Hence being able to match contact terms in the string scattering amplitudes with the appropriate field theory calculations gives a non-trivial verification that the Wess-Zumino action (1.3) must use the symmetric trace, at least at order λ^3. The general principle in constructing the action [8, 9] was consistency with T-duality. However, the symmetric trace is not required by T-duality, rather it was chosen to match the Matrix theory results for the linearized D0-brane couplings [13]. The appearance of the maximally symmetric trace in the Matrix theory calculations seems to be essentially a requirement of supersymmetry.

The same symmetrized trace was suggested by Tseytlin [29] in a discussion of the low energy gauge theory (with trivial background fields in the bulk). There it was shown that defining the non-abelian Born-Infeld action with this trace matched the known superstring results for the low energy scattering of gauge fields to fourth order in the field strengths. However, it was later shown that the symmetrized trace requires corrections at sixth order [30, 31]. These problems seem to be related to the ambiguity between covariant derivatives and field strengths in the non-abelian theory, in that the correction terms involve commutators of field strengths and so could be re-expressed in terms of covariant derivatives. It is quite probable that a fully consistent low energy action in the non-abelian theory will require the inclusion of interactions involving arbitrarily high derivatives of the gauge field strengths. Some progress in understanding the form of this action has recently been made [32] using ideas of non-commutative field theory [33].

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A. A useful integral

In this appendix, we evaluate the double integrals that appear in our calculations of the scattering amplitudes, i.e. eq. (3.11). The basic integral is

$$L_j \equiv (2i)^f \int_{-\infty}^{+\infty} dx_2 \int_{x_2}^{+\infty} dx_3\, (x_2-i)^a (x_2+i)^b (x_3-i)^c (x_3+i)^d (x_3-x_2)^e\,,$$

where

$$\begin{aligned}
a &\equiv 2k_2 \cdot p_4 + n^j_{24} \;=\; t + u + n^j_{24}\,, \\
b &\equiv 2k_2 \cdot p_4 + n^j_{25} \;=\; t + u + n^j_{25}\,, \\
c &\equiv 2k_3 \cdot p_4 + n^j_{34} \;=\; s + u + n^j_{34}\,, \\
d &\equiv 2k_3 \cdot p_4 + n^j_{35} \;=\; s + u + n^j_{35}\,, \\
e &\equiv 4k_3 \cdot k_2 + n^j_{32} \;=\; -2u + n^j_{32}\,, \\
f &\equiv p_4 \cdot D \cdot p_4 + n^j_{45} \;=\; -2s - 2t - 2u + n^j_{45}\,.
\end{aligned} \tag{A.1}$$

It is relatively straightforward to evaluate the integral over dx_3 leaving

$$\begin{aligned}
L_j = (2i)^f \int_{-\infty}^{+\infty} dx_2 \, \Bigg\{ & (x_2 - i)^{a+c}(x_2+i)^{b+d+e+1} \frac{\Gamma(-1-d-e)\Gamma(1+e)}{\Gamma(-d)} \times \\
& \times {}_2F_1\left(-c, 1+e, 2+d+e; \frac{x_2+i}{x_2-i}\right) + \\
& + (x_2-i)^{a+c+d+e+1}(x_2+i)^b \frac{\Gamma(-1-c-d-e)\Gamma(1+d+e)}{\Gamma(-c)} \times \\
& \times {}_2F_1\left(-d, -1-c-d-e, -d-e; \frac{x_2+i}{x_2-i}\right) \Bigg\}.
\end{aligned} \tag{A.2}$$

Using the following expansion of the hypergeometric function:

$${}_2F_1(\alpha_1, \alpha_2, \beta; z) = \frac{\Gamma(\beta)}{\Gamma(\alpha_1)\Gamma(\alpha_2)} \sum_{n=0}^{\infty} \frac{\Gamma(\alpha_1+n)\Gamma(\alpha_2+n)}{\Gamma(\beta+n)} \frac{z^n}{n!}\,,$$

one can write eq. (A.2) as

$$\begin{aligned}
L_j \;=\; & (2i)^f \frac{\Gamma(1+d+e)\Gamma(-d-e)}{\Gamma(-c)\Gamma(-d)} \times \\
& \times \sum_{n=0}^{\infty} \Bigg\{ -\frac{\Gamma(-c+n)\Gamma(1+e+n)}{n!\Gamma(2+d+e+n)} \int_{-\infty}^{+\infty} dx_2 (x_2-i)^{a+c-n}(x_2+i)^{b+d+e+1+n} + \\
& \quad + \frac{\Gamma(-d+n)\Gamma(-1-c-d-e+n)}{n!\Gamma(-d-e+n)} \times \\
& \quad \times \int_{-\infty}^{+\infty} dx_2 (x_2-i)^{a+c+d+e+1-n}(x_2+i)^{b+n} \Bigg\}.
\end{aligned}$$

In simplifying this expression, we have used the identity

$$\Gamma(-1-d-e)\Gamma(2+d+e) = -\Gamma(-d-e)\Gamma(1+d+e)\,.$$

Now the integrals over dx_2 have the general form

$$\begin{aligned}
\int_{-\infty}^{+\infty} dx (x-i)^A (x+i)^B &= -\frac{\pi(-i)^{2A}(2i)^{2+A+B}\Gamma(-1-A-B)}{\Gamma(-A)\Gamma(-B)} \\
&= \frac{(-i)^{2A}(2i)^{2+A+B}\sin(\pi A)\Gamma(1+A)\Gamma(-1-A-B)}{\Gamma(-B)}\,.
\end{aligned}$$

Using this result and the following identities:

$$\begin{aligned} a+b+c+d+e+f &= -3\,, \\ (-i)^{2(y-n)}\sin[\pi(y-n)] &= (-i)^{2y}\sin(\pi y)\,, \\ \Gamma(y)\Gamma(1-y) &= \frac{\pi}{\sin(\pi y)}\,, \end{aligned}$$

one finds

$$\begin{aligned} L_j =& -\frac{\pi\Gamma(-2-a-b-c-d-e)}{\sin[\pi(d+e)]\Gamma(-c)\Gamma(-d)}\times \\ &\times\sum_{n=0}^{\infty}\Bigg\{-(-i)^{2(a+c)}\sin[\pi(a+c)]\,\frac{\Gamma(-c+n)\Gamma(1+e+n)\Gamma(1+a+c-n)}{n!\Gamma(2+d+e+n)\Gamma(-1-b-d-e-n)}+ \\ &\qquad +(-i)^{2(a+c+d+e)}\sin[\pi(a+c+d+e)]\times \\ &\qquad \times\frac{\Gamma(-d+n)\Gamma(-1-c-d-e+n)\Gamma(2+a+c+d+e-n)}{n!\Gamma(-d-e+n)\Gamma(-b-n)}\Bigg\}. \end{aligned} \tag{A.3}$$

To rewrite this result in terms of the generalized hypergeometric function ${}_3F_2$, we begin by applying the identities:

$$\begin{aligned} \frac{\Gamma(1+a+c-n)}{\Gamma(-1-b-d-e-n)} &= -\frac{\sin[\pi(b+d+e)]\Gamma(2+b+d+e+n)}{\sin[\pi(a+c)]\Gamma(-a-c+n)}\,, \\ \frac{\Gamma(2+a+c+d+e-n)}{\Gamma(-b-n)} &= -\frac{\sin(\pi b)\Gamma(1+b+n)}{\sin[\pi(a+c+d+e)]\Gamma(-1-a-c-d-e+n)}\,. \end{aligned}$$

This allows eq. (A.3) to be expressed as

$$\begin{aligned} L_j =& \frac{\pi\Gamma(-2-a-b-c-d-e)}{\sin[\pi(d+e)]\Gamma(-d)\Gamma(-c)}\times \\ &\times\sum_{n=0}^{\infty}\Bigg\{-(-i)^{2(a+c)}\sin[\pi(b+d+e)]\times \\ &\qquad\times\frac{\Gamma(-c+n)\Gamma(1+e+n)\Gamma(2+b+d+e+n)}{n!\Gamma(2+d+e+n)\Gamma(-a-c+n)}+ \\ &\qquad +(-i)^{2(a+c+d+e)}\sin(\pi b)\frac{\Gamma(-d+n)\Gamma(-1-c-d-e+n)\Gamma(1+b+n)}{n!\Gamma(-d-e+n)\Gamma(-1-a-c-d-e+n)}\Bigg\}. \end{aligned} \tag{A.4}$$

Now using the definition of ${}_3F_2$

$${}_3F_2(\alpha_1,\alpha_2,\alpha_3;\beta_1,\beta_2;1) = \frac{\Gamma(\beta_1)\Gamma(\beta_2)}{\Gamma(\alpha_1)\Gamma(\alpha_2)\Gamma(\alpha_3)}\sum_{n=0}^{\infty}\frac{\Gamma(\alpha_1+n)\Gamma(\alpha_2+n)\Gamma(\alpha_3+n)}{\Gamma(\beta_1+n)\Gamma(\beta_2+n)n!}$$

one may write eq. (A.4) as

$$\begin{aligned} L_j =& -\Gamma(-2-a-b-c-d-e)\times \\ &\times\Bigg\{(-i)^{2(a+c)}\sin[\pi(b+d+e)]\,\frac{\Gamma(-1-d-e)\Gamma(1+e)\Gamma(2+b+d+e)}{\Gamma(-d)\Gamma(-a-c)}\times \\ &\quad\times{}_3F_2(-c,1+e,2+b+d+e;2+d+e,-a-c;1)+ \\ &\quad+\ (-i)^{2(a+c+d+e)}\sin(\pi b)\frac{\Gamma(1+d+e)\Gamma(1+b)\Gamma(-1-c-d-e)}{\Gamma(-c)\Gamma(-1-a-c-d-e)}\times \\ &\quad\times{}_3F_2(-d,-1-c-d-e,1+b;-d-e,-1-a-c-d-e;1)\Bigg\}, \end{aligned} \tag{A.5}$$

where the following identities have been used

$$\begin{aligned} \sin[\pi(d+e)]\,\Gamma(2+d+e) &= \frac{\pi}{\Gamma(-1-d-e)}, \\ \sin[\pi(d+e)]\,\Gamma(-d-e) &= -\frac{\pi}{\Gamma(1+d+e)}. \end{aligned}$$

Given the definition of the exponents (A.1), eq. (A.5) reduces to the result given in eq. (3.13).

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